

Probing the creatable character of perturbed Friedmann-Robertson-Walker universes

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Abstract

We discuss whether some perturbed Friedmann-Robertson-Walker (FRW) universes could be creatable, i. e., could have vanishing energy, linear momentum and angular momentum, as it could be expectable if the Universe arose as a quantum fluctuation. On account of previous results, the background is assumed to be either closed (with very small curvature) or flat. In the first case, fully arbitrary linear perturbations are considered; whereas in the flat case, it is assumed the existence of: (i) inflationary scalar perturbations, that is to say, Gaussian adiabatic scalar perturbations having an spectrum close to the Harrison-Zel'dovich one, and (ii) arbitrary tensor perturbations. We conclude that, any closed perturbed universe is creatable, and also that, irrespective of the spectrum and properties of the inflationary gravitational waves, perturbed flat FRW universes with standard inflation are not creatable. Some considerations on pre-inflationary scalar perturbations are also presented. The creatable character of perturbed FRW universes is studied, for the first time, in this paper.

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I. INTRODUCTION

In [1] (see also the addenda in [2]), for a wide set of non asymptotic Minkowskian space-times, it was uniquely determined when one of these space-times can be said to have vanishing energy, vanishing linear 3-momentum and vanishing angular 4-momentum, that is, vanishing energy and momenta. These vanishing values could be expected in the case of a universe which rose from a quantum vacuum fluctuation [3][4]. In [1], universes of this kind, with vanishing energy and momenta, were called ‘creatable universes’.

It is very well known that, whatever the energy-momentum complex may be, the definition of energy and momenta of the universe is strongly dependent on the coordinate system. One must then stress the uniqueness reached [1] in the characterization of the family of creatable universes, within the above wide set. This uniqueness has been reached by using some physical criteria to select the appropriate coordinate systems. See [5] as an example of a more mathematical criterion, which leads the author to use conformally flat coordinates in the space-time, when it is possible.

In order to look for convenient physical criteria to select the appropriate coordinates, one should assume that the proper energy and momenta of any space-time representing the universe is conserved in time. Taking into account that the Universe is supposed to embrace everything, fluxes of energy and momenta going out of such an entity are not possible and, consequently, the above assumption about time conservation seems actually appropriate. Such an assumption only has a clear physical meaning if it is referred to a physical (proper) and universal (synchronous) time. In other words, the conserved energy and momenta of a universe should be calculated by using Gauss coordinates, which involve the required time coordinate. Energy and momenta would be then conserved in terms of Gauss time coordinates. Because of this conservation, no global Gauss coordinates are necessary in practise to calculate energy and momenta. Actually, Gauss coordinates defined in the elementary vicinity of a generic space-like 3-surface suffice.

Given a space-time, there are plenty of different Gauss coordinate systems. Thus, we will need to make sure that the characterization of any perturbed FRW universe as creatable or non creatable is independent on the Gauss coordinate used. We will consider various of these perturbed universes and, then, in order to see if they are creatable, we will apply different strategies, inspired by [1], which are adapted to each particular case. Nevertheless,

any of these strategies will obey the following protocol: (i) take any space-like 3-surface, Σ_3 , and build the corresponding Gauss coordinates in its elementary vicinity, (ii) look for new Gauss coordinates leading to an ‘instantaneous’ 3-space metric, $dl_0^2 \equiv dl_0^2(t = t_0)$, which explicitly exhibits its conformal character on the boundary, Σ_2 , of Σ_3 (these coordinates always exist; see [1]) and, (iii) calculate the energy and momenta of the universe in the resulting coordinate system. Despite the still remaining freedom in the election of these coordinates, it can be seen that the calculated values of energy and momenta are unique for the different cases we consider in the present paper. In all these cases, if these energy and momenta vanish, the universe is creatable, since it can be directly seen that they vanish irrespective of both the selected space-like 3-surface, Σ_3 , and the chosen time, t_0 (energy and momenta conservation). This will be verified at the end of each particular case.

It has sometimes been argued (see, e. g., [5]) that, in a space-time which is not asymptotically flat, the global energy and momenta would have no physical meaning, since these quantities could never be measured; however, it is also claimed that the energy and momenta of any part of this space-time have physical meaning. This double claim is not fully consistent; in fact, if energy and momenta have physical meaning for any part of the space-time, the corresponding global quantities should be interpreted as the limit of the physically significant energy and momenta of the parts, as they grow to fill the entire space-time.

By using the Weinberg energy-momentum complex [10] and the above protocol, it was proved that the closed and flat Friedmann-Robertson-Walker (FRW) universes ($K = 0, +1$) are creatable, whereas the open version ($K = -1$) is not [1]. This is in agreement with most of the papers on the subject, but not with all of them (see again [5]). Similar conclusions were obtained by using very different methods. For example, in papers [6] and [7], it was proved that the tunneling amplitude for creation from ‘nothing’ is finite in the case of closed cosmological backgrounds and for a flat De Sitter unperturbed universe. These two papers concluded that the tunneling amplitude vanishes in the open case. These conclusions were obtained in the framework of particular FRW models for tunneling. Realistic universes with perturbations were not considered at all. The main goal of the present paper is the study of perturbed FRW universes without any calculation of transition amplitudes. Instead of these calculations, we use the above protocol to look for creatable universes (those with vanishing energy and momenta) without modelling any quantum tunneling.

The observations seem to indicate that we live in a perturbed FRW universe and, con-

sequently, if the above protocol makes sense, at least one of the perturbed FRW universes should be creatable. Our results confirm this expectation, since we have proved that any perturbed closed universe is creatable. The perturbed $K = -1$ universe does not need to be considered here because the corresponding background is not creatable. The perturbed flat case is studied in detail along the paper. It is not creatable, under very general conditions strongly supported by current observations.

This paper is organized as follows: In Sec. II, we consider absolutely arbitrary linear perturbations in the case $K = +1$. The case $K = 0$ is studied in Sec. III. The total energy due to the scalar perturbations arising in standard inflationary models is calculated in Sec. III A (it is infinite). Sec. III B contains the calculation of the same quantity in the case of fully arbitrary gravitational waves (it vanishes). Finally, in Sec. IV, we summarize our main conclusions and present a general discussion, including comparisons between our results and methods and those of previous papers dealing with quantum creation from 'nothing' (see references [6]–[9]).

Some of these results have been briefly presented, with no calculations, in the meeting ERE-2007 [11]

Let us finish this section with some words about notation. Units are chosen in such a way that the speed of light is $c = 1$. The gravitational constant, the scale factor, and the index of the 3-space curvature are denoted G , a , and K , respectively. Symbols t and τ stand for the Gauss and the conformal times ($ad\tau = dt$) and, finally, the unit vector $\hat{\mathbf{k}}$ and the modulus k define a generic vector $\mathbf{k} = k\hat{\mathbf{k}}$ in momentum space.

II. THE CASE OF A PERTURBED CLOSED FRW UNIVERSE

The line element of the closed FRW space-time can always be written in the form:

$$ds^2 = -dt^2 + dl^2, \quad dl^2 = \frac{a^2(t)}{\left(1 + \frac{K}{4}r^2\right)^2} \delta_{ij} dx^i dx^j, \quad r^2 \equiv \delta_{ij} x^i x^j \quad (1)$$

with $K = +1$. Here, x^i and t are global Gauss coordinates. Moreover, the 3-space metric exhibits a conformally flat form everywhere.

Now, let us consider the case of a perturbed closed FRW universe with scalar and tensor perturbations [12]. Then, in the synchronous gauge (see the Appendix), conditions $g_{00} = -1$, $g_{i0} = 0$ are satisfied. Hence, in this gauge, Gauss coordinates are used. In terms of them,

the 3-space metric, dl^2 , reads as follows:

$$dl^2 = \frac{a^2(t)}{(1 + \frac{1}{4}r^2)^2}(\delta_{ij} + h_{ij})dx^i dx^j, \quad (2)$$

where the $h_{ij}(t, x^i)$ functions are such that $h_{ij} \ll 1$.

Let it be a particular space-like 3-surface, $t = t_0$, and particularize the above 3-space metric on this 3-surface, that is, consider $dl(t = t_0)^2 \equiv dl_0^2$. This instantaneous 3-space metric is a conformally flat 3-metric on the boundary 2-surface, Σ_2 , of $t = t_0$ [1]. Then, among the different Gauss coordinate systems, one always can select some ones such that dl_0^2 on the boundary Σ_2 , say $dl_0^2|_{\Sigma_2}$, shows explicitly its conformally flat character. According to the protocol displayed in Sec. I, which can be used here to calculate the energy and momenta of perturbed closed universes, we can pick up any one of these last coordinate systems to compute the corresponding values of the energy and momenta of our perturbed universe. We will subsequently see that these values are unique.

Then, according to [10], the energy, P^0 , the 3-momentum, P^i , and the 4-angular momentum, (J^{jk}, J^{0i}) , of the universe are:

$$P^0 = \frac{1}{16\pi G} \int (\partial_j g_{ij} - \partial_i g) d\Sigma_{2i}, \quad (3)$$

$$P^i = \frac{1}{16\pi G} \int (\dot{g}\delta_{ij} - \dot{g}_{ij}) d\Sigma_{2j}, \quad (4)$$

$$J^{jk} = \frac{1}{16\pi G} \int (x_k \dot{g}_{ij} - x_j \dot{g}_{ki}) d\Sigma_{2i}, \quad (5)$$

$$J^{0i} = P^i t - \frac{1}{16\pi G} \int [(\partial_k g_{kj} - \partial_j g)x_i + g\delta_{ij} - g_{ij}] d\Sigma_{2j}, \quad (6)$$

where $\dot{g}_{ij} \equiv \partial_t g_{ij}$, $g \equiv \delta_{ij} g_{ij}$, and where $d\Sigma_{2i}$ stands for the integration element on Σ_2 . In the present case, the 2-surface Σ_2 is $r = \infty$.

Now, it is obvious that the energy and momenta of our perturbed closed FRW universe must vanish; in fact, according to Eq. (2), all the integrands in the above 2-surface integrals go at least like $1/r^4$ as r tends to ∞ . (Notice that h_{ij} cannot grow indefinitely with r , otherwise we would not have $h_{ij} \ll 1$ everywhere). Of course, this asymptotic behavior, in terms of $1/r$, is valid for all Gauss coordinates which preserve the perturbed character of the metric Eq. (2), and so, whatever be the time parameter t_0 , it is valid for the particular Gauss coordinate system where the 3-metric, dl_0^2 , shows explicitly its conformal flat character on

Σ_2 . Evidently, such a behavior of the integrands implies straightforward that all the integrals in Eqs. (3)–(6) vanish irrespective of the above Gauss coordinate used. Consequently, we can state that, whatever the properties of the perturbations may be, perturbed closed FRW universes are creatable.

III. THE CASE OF A PERTURBED FLAT FRW UNIVERSE

In the case of a flat background, we cannot invoke the strong decaying of the 3-space metric, as r tends to ∞ , to conclude that energy and momenta vanish; namely, arguments similar to those of the last section do not apply. Furthermore, one could erroneously think that, in the flat case, the vanishing of the energy would come straightforward from the cosmological principle; namely, from the assumed statistical homogeneity when averaging on large enough volumes. Actually, as we will see in detail along this section, the values of the energy and momenta of the universe depend on both the statistical character and the spectra of the perturbations.

A. Scalar perturbations

Let us first consider the case of scalar perturbations [12] in a flat FRW universe. In any synchronous gauge (Gauss coordinates), the perturbed 3-space metric is $g_{ij} = a^2(\tau)(\delta_{ij} + h_{ij})$ and, according to [13], the metric perturbation, h_{ij} , can be expanded in scalar harmonics (plane waves) as follows:

$$h_{ij}(\mathbf{x}, \tau) = \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} h_{ij}(\mathbf{k}, \tau) = \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} [\hat{\mathbf{k}}^i \hat{\mathbf{k}}^j h(\mathbf{k}, \tau) + (\hat{\mathbf{k}}^i \hat{\mathbf{k}}^j - \frac{1}{3}\delta_{ij}) 6\eta(\mathbf{k}, \tau)] , \quad (7)$$

where two functions, $h(\mathbf{k}, \tau)$ and $\eta(\mathbf{k}, \tau)$, defined in momentum space, have been introduced. Notice that, in order to have a real value for h_{ij} , functions h and η must satisfy the conditions $h(-\mathbf{k}) = -h^*(\mathbf{k})$ and $\eta(-\mathbf{k}) = -\eta^*(\mathbf{k})$.

According to the above protocol to calculate energy and momenta, we must use a new Gauss coordinate system, in which, the transformed components of the instantaneous 3-space metric, $g'_{ij}(t = t_0)$, show explicitly its conformal flat character on the boundary $r = \infty$. In the present case, we can do more than this since our instantaneous 3-space metric is a conformally flat metric everywhere on the space-like 3-surface $t = t_0$ (not only on the 2-surface $r = \infty$). In order to prove this statement, let us work in the \mathbf{k} -space. Given a

\mathbf{k} -value, let us consider the infinitesimal coordinate transformation

$$x^i = x'^i + e^i(\mathbf{x}, \mathbf{k}), \quad (8)$$

which leads to the following equation:

$$h'_{ij}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} = h_{ij}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + \partial_i e_j + \partial_j e_i. \quad (9)$$

If we then choose $e^i(\mathbf{x}, \mathbf{k}) = f(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\hat{\mathbf{k}}^i$, where $f(\mathbf{k})$ is defined by the relation

$$(h + 6\eta + 2a^2k^2f)_{t=t_0} = 0, \quad (10)$$

equation (9) leads to the following new Fourier component $h'_{ij}(\mathbf{k})$ for $t = t_0$

$$h'_{ij}(\mathbf{k})_{t=t_0} \equiv h'_{ij}(\mathbf{k})_0 = -2\eta_0(\mathbf{k})\delta_{ij}, \quad (11)$$

where $\eta_0(\mathbf{k}) \equiv \eta(\mathbf{k}, t = t_0)$. Equation (11) implies that the instantaneous metric $g_{ij}(t = t_0)$ is conformally flat everywhere on the 3-space $t = t_0$, as we wanted to prove.

To calculate the energy, we must insert the new components of the 3-space metric $g'_{ij}(t = t_0) = a_0^2[\delta_{ij} + h'_{ij}(t = t_0)]$ in the integral of Eq. (3), which must be performed on the 2-surface $r = \infty$. This integral is to be calculated in the new coordinates x'^i ; however, in practice, the old coordinates, x^i , can be used in first order calculations; in fact, since the integrand in Eq. (3) trivially vanishes for the background metric (a flat FRW universe), this integrand is a first order quantity in the perturbed universe and, consequently, the integral can be evaluated irrespective of the coordinate system (the differences between the integrands in the old and new coordinates are second order quantities to be neglected in any linear approach). In short, to first order, metric perturbations and its derivatives can be calculated in terms of the old coordinates, and the old 2-surface element, $d\Sigma_{2i}$, can be used instead of the new one. Thus, according to Eq. (11), the total energy P^0 can be written as follows:

$$P^0 = \lim_{r \rightarrow \infty} \frac{a_0^2 r^2}{8\pi G} \int d\Omega n_i \partial_i \int dk^3 \eta_0 e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (12)$$

where $d\Omega$ is the solid angle element in spherical coordinates, i. e., $d\Omega = \sin\theta d\theta d\phi$, and where $n_i \equiv x^i/r$.

It is noticeable that, according to the above expression of P^0 , the energy of the universe does not depend on the function h in Eq. (7). It only depends on the function η , that is, on the traceless part of the perturbed metric (7).

We assume statistical isotropy, which characterizes cosmological processes as, e. g., standard inflation. Since there are no privileged directions, the power spectrum of $\eta(\mathbf{k})$, namely, the function $P_\eta(\mathbf{k}) = \langle |\eta(\mathbf{k})|^2 \rangle$ is independent on $\hat{\mathbf{k}}$. It only depends on k . Three k -intervals can be distinguished (see the Appendix). The interval $(0, k_{min})$ involving pre-inflationary perturbations with some unknown power spectrum $P_{\eta 1}(\mathbf{k})$, the (k_{min}, k_{max}) interval with inflationary perturbations evolving outside the horizon, whose spectrum is $P_{\eta 2}(\mathbf{k})$, and the interval (k_{max}, ∞) with inflationary perturbations which have reentered the horizon; in this case, the spectrum is denoted $P_{\eta 3}(\mathbf{k})$.

Inflationary perturbations are all Gaussian in the evolution period under consideration (before entering in nonlinear processes, see the Appendix); hence, if pre-inflationary perturbations are also assumed to be Gaussian (the non Gaussian case is discussed below), functions η and h , as well as the Fourier transform of any physical quantity (density contrasts, pressure, and so on) are complex functions with random phases [14] for $0 < k < \infty$ and, consequently, the η values necessary to calculate P^0 –from Eq. (3)– can be written in the form

$$\eta(\mathbf{k}) = \frac{1}{\sqrt{2}} \left[P_\eta(k) \right]^{1/2} \left[y_1(\mathbf{k}) + iy_2(\mathbf{k}) \right] , \quad (13)$$

where $y_1(\mathbf{k}) + iy_2(\mathbf{k})$ are the phases, which must be assigned taking into account the relation $\eta(-\mathbf{k}) = -\eta^*(\mathbf{k})$ and the relations:

$$\langle y_1(\mathbf{k}) \rangle = \langle y_2(\mathbf{k}) \rangle = 0 , \quad (14)$$

$$\langle y_1(\mathbf{k}) y_2(\mathbf{k}') \rangle = 0 , \quad (15)$$

$$\langle y_n(\mathbf{k}) y_n(\mathbf{k}') \rangle = \delta^3(\mathbf{k} - \mathbf{k}') , \quad (16)$$

where index n run from 1 to 2, vectors \mathbf{k} and \mathbf{k}' are arbitrary momenta, and the averages are to be performed in the set of the universe realizations. In the last equation, δ^3 is the three dimensional Dirac δ -distribution. Equations (14)–(16) ensure (central limit theorem) that the distribution of P^0 values corresponding to all the possible realizations of the universe is Gaussian.

By using the relation

$$\int d\Omega n_i e^{i\mathbf{k} \cdot \mathbf{x}} = \frac{4\pi i}{kr} \left(\frac{\sin kr}{kr} - \cos kr \right) \hat{\mathbf{k}}_i , \quad (17)$$

which can be easily obtained, plus Eqs. (12) and (13), one easily finds:

$$P^0 = -(a_0^2/2G) \lim_{r \rightarrow \infty} r \sum_{j=1}^3 \int_j d^3k \left[P_{\eta j}(k) \right]^{1/2} \left(\frac{\sin kr}{kr} - \cos kr \right) y_1(\mathbf{k}) , \quad (18)$$

where the index j number the three intervals defined above. The volume integrals in momentum space are extended to the regions $k < k_{min}$, $k_{min} < k < k_{max}$, and $k > k_{max}$, in cases $j=1$, $j=2$, and $j=3$, respectively. In order to derive the last equation, we have used the relation $\eta(-\mathbf{k}) = -\eta^*(\mathbf{k})$ and, consequently, the energy P^0 appears to be real valued as it must be. It is also important that, as a result of the same relation, quantity P^0 only depends on the random variable $y_1(\mathbf{k})$; whereas $y_2(\mathbf{k})$ becomes fully irrelevant in our calculations.

Taking into account Eq. (18) and the fact that $\langle y_1(\mathbf{k}) \rangle$ vanishes, one easily finds the relation $\langle P^0 \rangle = 0$. Of course, the average is performed for all the possible realizations of the universe, which correspond to distinct $y_1(\mathbf{k})$ values but to the same power spectrum. Since the phases have been chosen in such a way that the P^0 values are normally distributed, we must now calculate the variance, $\langle (P^0)^2 \rangle$, which fully characterize this normal (Gaussian) distribution with vanishing mean. From Eq. (18), one easily finds a formula for $(P^0)^2$ which is the addition of six different terms. Three of them involve products of integrals corresponding to distinct j -values (hereafter crossed terms). Since two \mathbf{k} -vectors corresponding to distinct j values cannot coincide, equation (16) implies that, after averaging, the crossed terms vanish. From the remaining three terms plus Eq. (16) one easily obtains the following variance:

$$\langle (P^0)^2 \rangle = (a_0^4/4G^2) \lim_{r \rightarrow \infty} r^2 \sum_{j=1}^3 \int_j d^3k P_{\eta j}(k) \left(\frac{\sin kr}{kr} - \cos kr \right)^2 . \quad (19)$$

Each of the three terms of this last formula is positive (it is the square of one of the three integrals $j = 1, 2, 3$) and, consequently, if one of them diverges, the variance $\langle (P^0)^2 \rangle$ diverges.

In the case of inflationary adiabatic scalar perturbations which evolve outside the horizon in the radiation dominated era ($k_{min} < k < k_{max}$), the solution of the linearized Einstein field equations can be found in Ma and Bertschinger [13] (see the Appendix for more details). In the synchronous gauge, function η evolves as follows:

$$\eta = C + \frac{3}{4} \frac{D}{k\tau} + \alpha C k^2 \tau^2 ; \quad (20)$$

in this equation, α is a pure number (see the Appendix), whereas C and D are dimensionless integration constants. That is to say, C and D cannot depend on time, but they may depend

on \mathbf{k} . Notice that here, as well as in [13], the components of \mathbf{x} and \mathbf{k} are dimensionless, since they are comoving coordinates and their associated momenta. We see that η is the addition of three different modes: one of them grows like τ^2 , another one is constant, and the third one decreases like τ^{-1} .

Let us now study the behavior of the integrals involved in Eq. (19) as r tends to ∞ . We begin with one of the contributions to the $j = 2$ integral (inflationary Gaussian perturbations with super-horizon sizes). It is the contribution due to the constant mode $C(\mathbf{k})$ in Eq. (20). This contribution is hereafter denoted $\langle(\tilde{P}^0)^2\rangle$. The spectrum $P_c(k)$ of the constant mode is defined by the relation

$$\left[P_c(k)\right]^{1/2} = Ak^n, \quad (21)$$

where A is a normalization constant and the spectral index is $n = -(3/2) - \beta$. According to Eq. (A.10) in the Appendix, constant $\beta = (1 - n_s)/2$ is positive and small as compared to unity. From the spectrum (21), one easily finds the following variance:

$$\langle(\tilde{P}^0)^2\rangle = (\pi a_0^4 A^2 / G^2) \lim_{r \rightarrow \infty} r^{2+2\beta} \int_{rk_{min}}^{rk_{max}} dy \left(\frac{\sin y}{y} - \cos y \right)^2 / y^{1+2\beta}, \quad (22)$$

where $y \equiv kr$. For any k , the new variable y tends to infinity as r does. On account of this fact, it is easily proved that the integral in Eq. (22) goes just like the power $r^{-2\beta}$ as r tends to infinity. Hence, $\langle(\tilde{P}^0)^2\rangle$ diverges as r^2 .

Let us now consider the growing and decaying terms appearing in Eq. (20). It is obvious that the contributions of these time dependent terms to the energy cannot compensate the infinite value of $\langle(\tilde{P}^0)^2\rangle$ corresponding to the constant mode, which implies that the total contribution to $\langle(P^0)^2\rangle$ due to the $j = 2$ k -interval cannot become finite. More precisely, from Eq. (20) we find six distinct terms contributing to the $j = 2$ integral. The term used in the previous calculation does not depend on time, whereas the remaining ones are time dependent. By this reason, the variance $\langle(\tilde{P}^0)^2\rangle$ is proportional to a^4 for the time independent mode, whereas it exhibits other time dependences in the remaining cases. In all, compensation of the resulting terms (with distinct time evolutions) to give a conserved finite total energy is not possible, which ensures that the entire contribution $j = 2$ to $\langle(P^0)^2\rangle$ is infinite. Moreover, this positive contribution cannot be compensated by those of the $j = 1$ and $j = 3$ cases to give, finally, a finite global value for $\langle(P^0)^2\rangle$, the reason being that, as explained above, the $j = 1$ and $j = 3$ contributions are also positive. All in all, the following

relation holds:

$$\langle (P^0)^2 \rangle = \infty \quad (23)$$

and, consequently, we have a Gaussian statistical distribution of P^0 values with zero mean and infinite variance. Since the Gaussian probability density is

$$P(P^0) = \frac{1}{\sqrt{2\pi\langle (P^0)^2 \rangle}} e^{-(P^0 - \langle P^0 \rangle)^2 / 2\langle (P^0)^2 \rangle}, \quad (24)$$

it is evident that, in our case, the probability of any particular finite value of P^0 vanishes and, as a consequence, we can say that, in the flat perturbed FRW universe under consideration, the contribution of the inflationary scalar perturbations to the total energy is infinite. Nevertheless, in the standard inflationary paradigm, there are also tensor perturbations, whose contribution to the total energy of a flat perturbed universe is calculated in next section.

Let us now discuss the case of non-Gaussian pre-inflationary perturbations. These perturbations are assumed to be generated in some process fully independent on inflation and, as explained in the Appendix, they are assumed to be significant only in the interval $(0, k_{min})$; hence, Eqs. (16) are satisfied except for pairs of vectors \mathbf{k} and \mathbf{k}' whose modulus are both inside the interval $(0, k_{min})$. Therefore, there are no crossed terms in the development of $\langle (P^0)^2 \rangle$ (see above). As in Eq. (19), the variance $\langle (P^0)^2 \rangle$ is the addition of three positive terms. Terms $j = 1$ and $j = 2$ have the same form as in the Gaussian case, whereas the term $j = 3$ would be different. In this situation, the term $j = 2$ diverges (same discussion as above) and, consequently, Eq. (23) holds (which strongly suggests a non creatable universe). However, the distribution of P^0 values is not Gaussian in this case, and its probability density should be calculated for each particular non Gaussian model. Since Eq. (24) does not apply, the meaning of an infinite variance is now less clear. By this reason, in order to properly prove that flat universes with standard inflation are not creatable, we prefer a general argument, which proves that, if $\langle (P^0)^2 \rangle$ diverges in the interval (k_{min}, ∞) , namely, if the total energy of the inflationary Gaussian scalar perturbations is infinite, the energy of the universe cannot vanish whatever the properties of the non-Gaussian pre-inflationary perturbations may be; namely, the universe is not creatable. This is trivially proved taking into account that, in any admissible universe, the total energy must be a conserved quantity (see Sec. I) which vanishes in the creatable case. In fact, a vanishing energy after inflation is only possible if the pre-inflationary energy is infinite and it exactly compensates the infinite

energy associate to the scalar inflationary perturbations. However, such a universe would have an infinite energy before inflation and a vanishing one after this process, which is not compatible with the required energy conservation.

Now, before ending this Section, we raise some comments about the consistence of the main result: the infinite value of the energy we have found.

First of all, although the infinite energy has been formally obtained for a given value of t_0 and a certain Σ_3 , it is obvious, by following the implementation of the protocol we have used, that this t_0 value and the choice of Σ_3 are both arbitrary.

Finally, we could perform a conformal coordinate transformation and still retain the explicit conformal flat form of the 3-space metric on Σ_3 (remember that, according to Eq. (11), in the present case, the 3-space metric is a conformally flat one all over Σ_3). But, trivially, the above infinite value of the energy does not depend on the conformal transformations which are pertinent here: those belonging to the translation, rotation, and dilatation subgroups of the conformal group. For all these coordinate transformations the energy remains infinite.

B. Tensor perturbations

In this section we are concerned with tensor perturbations evolving in a flat background. In such a case, the instantaneous 3-space line element is: $g_{ij} = a_0^2(\delta_{ij} + h_{ij}^T)$. In Fourier space, we can write

$$h_{ij}^T(\mathbf{k}) = H(\mathbf{k}, \tau_0)\epsilon_{ij}(\hat{\mathbf{k}}), \quad (25)$$

where the quantities ϵ_{ij} satisfy the conditions given in the Appendix.

Let us calculate the energy, P_T^0 , of these tensor perturbations. According to our protocol, calculations must be performed in a new coordinate system, in which the instantaneous 3-dimensional metric explicitly exhibits its conformally flat character on the boundary 2-surface $r = \infty$. For each value of \mathbf{k} , let us consider a coordinate transformation of the form (8). Functions $e^i(\mathbf{x}, \mathbf{k})$ must be chosen in such a way that, in the new coordinates x'^i , the line element on the surface $r = R$, where R is an arbitrary constant (at the end of our calculations, this constant will tend to ∞), has the form

$$h_{ij}^{T'}(\mathbf{x})\Big|_{r=R} = f(\mathbf{n}, \mathbf{k})e^{iR\mathbf{k}\cdot\mathbf{n}}\delta_{ij}, \quad (26)$$

where $h^{T'}_{ij}(\mathbf{x}) = h^{T'}_{ij}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$ is the metric perturbation in the space-like hypersurface $\tau = \tau_0$ corresponding to the fixed mode \mathbf{k} , and where function $f(\mathbf{n}, \mathbf{k})$ is the conformal factor of the metric on $r = R$. As it has been said above, there always exists a family of coordinate systems in which Eq. (26) is satisfied [1]. The energy and momenta can be calculated in any of these coordinates. With the appropriate e_i functions, from Eq. (8), one easily finds the relation

$$h^{T'}_{ij}(\mathbf{x}) = h^T_{ij}(\mathbf{x}) + \partial_i e_j + \partial_j e_i, \quad (27)$$

which is equivalent to Eq. (9).

The energy, P^0_T , is calculated by using Eq. (3) and the metric perturbation components $h^{T'}_{ij}(\mathbf{x}')$; nevertheless, in the linear approach we are using (see Sec. III A for details), the following approximations can be performed: (i) write these components in terms of the old coordinates x^i , (ii) perform the derivatives with respect to x^i , and (iii) use the old 2-surface element, $d\Sigma^i_2$, instead of $d\Sigma'^i_2$. Hence, we can write

$$P^0_T = (a_0^2/16\pi G) \int \partial_j \left[h^{T'}_{ij}(\mathbf{x}) - \partial_i h^{T'}(\mathbf{x}) \right] d\Sigma^i_2. \quad (28)$$

Taking into account Eq. (27), this last equation can be rewritten as follows:

$$P^0_T = (P^0_T)_H + (a_0^2/16\pi G) \int \left[\partial_j (\partial_i e_j + \partial_j e_i) - 2\partial_i \partial_k e_k \right] d\Sigma^i_2, \quad (29)$$

where $(P^0_T)_H$ is the energy corresponding to the first term of the r.h.s. of Eq. (27), whose Fourier transform, $h^T_{ij}(\mathbf{k})$, is given by Eq. (25). As a result of the conditions satisfied by the quantities ϵ_{ij} (see the Appendix), it is easily proved that the term $(P^0_T)_H$ vanishes. Hence,

$$P^0_T = (a_0^2/16\pi G) \int (\partial_j \partial_j e_i - \partial_i \partial_j e_j) d\Sigma^i_2, \quad (30)$$

and, finally, the Gauss theorem allow us to write the surface integral in the last equation as a vanishing volume integral whatever the functions e_i may be and, in particular, for the functions leading to Eq. (26). Hence, we have proved that the energy associated to any distribution of gravitational waves (propagating in a flat universe) vanishes; namely, the equation

$$P^0_T = 0. \quad (31)$$

is satisfied.

Therefore, according to our protocol, we conclude that the energy P^0_T due to tensor perturbations of a flat FRW vanishes. Notice that this conclusion does not depend either

on the spectrum $P_T(k)$ of the tensor perturbations (see the Appendix), or on the statistical character of the distribution of these perturbations.

Notice again that similarly to what has been explained to the end of Sec. III A, the resulting vanishing energy does neither depend on the chosen value of the t_0 parameter, nor on the choice of Σ_3 .

Since the energies due to scalar and tensor perturbations add, realistic perturbed universes including inflationary scalar modes have in all an infinite energy and so are not creatable. The presence of arbitrary tensor perturbation (zero energy) is irrelevant. Coming back to the end of Sec. III A, we see now that the energy of the scalar perturbed closed FRW universes is infinite irrespectively of, not only the rotation, traslation and dilatation groups, but actually on any infinitesimal coordinate transformation.

IV. CONCLUSIONS AND DISCUSSION

Our main conclusion is that perturbed flat FRW universes, including arbitrary tensor perturbations, and the adiabatic Gaussian scalar ones generated during standard inflation, have an infinite energy which is due to the scalar perturbations (see Secs. III A and III B). Since the total energy does not vanish, perturbed flat universes are not creatable, at least, in the framework of the standard inflationary paradigm, which appears to be compatible with most current observations. This conclusion implies that, among the perturbed FRW universes undergoing ordinary inflation, only the closed ones are creatable and, consequently, the slightly inhomogeneous universe where we live should be closed.

It is generally believed that, in classical terms, there is no any way to decide if our universe is flat or closed, at least, if the curvature is small enough. An exception can be found in [15], where it is claimed that the *spiral geodesic effect* could be used to decide, observationally, whether we live either in a flat or a closed perturbed FRW universe. Another different method to distinguish between flat and closed perturbed universes arises from this paper. It is not directly based on observations. In our case, the creatable character of closed, and flat models compatible with observations, is studied according to the protocol described in Sec. I. The closed universes are creatable whatever the linear perturbations may be. This is a robust conclusion which privileges closed models against the flat ones. In the $K = 0$ case, we have studied the most accepted model based on standard inflation. It does not

appear to be creatable; nevertheless, this conclusion is not valid whatever the perturbations may be. On the contrary, it is based on some assumptions and, consequently, it must be revised if some of such assumptions are modified in future. Particular attention deserve our hypothesis about: (i) the statistical isotropy of the universe and, (ii) the adiabatic and Gaussian character of any scalar perturbation in the post-inflationary era for $k_{min} < k < \infty$. Statistical isotropy has been recently questioned [19] [20], and isocurvature and (or) non Gaussian perturbations (based on cosmic strings, pre-inflationary processes and, so on) are not completely forbidden for $k_{min} < k < \infty$. Perturbed flat universes violating condition (i), and (or) condition (ii), and (or) any other possible condition, require particular studies if one wants to probe its creatable character.

The subject of the creatable character of the (non perturbed) FRW universes has been considered in two papers in the first eighties [6][7] (there is also related work in [8] [9]). In [6][7], the authors discussed the possibility that the Universe could have arisen by quantum tunneling from ‘nothing’. In [7], a cosmological model is proposed in which the Universe is created by quantum tunneling from ‘nothing’ into a particular closed FRW model. Furthermore, in [6], the authors find that within the context of FRW models, only the spatially closed and the flat de Sitter universes can originate in this manner, because they find that a finite tunneling amplitude exists only from initial spaces with finite three-volume (on the Euclidean section).

In the absence of perturbations, the results in the present paper essentially agree with those of [6]–[9], since we find with many other authors (including paper [1]) that the closed and flat FRW models have vanishing energy and momenta and so, according to our terminology, this kind of universes would be creatable. Nevertheless, the method and the scopes of our work are very different from those of papers [6]–[9].

First of all, in these papers, the authors considered precise quantum mechanisms to originate our classical universe from a quantum one; however, we assume that the produced classical universes must have vanishing energy and momenta and, then, we apply a definite protocol to decide whether a given classical universe (in the present case, some perturbed FRW models) has or not vanishing energy and momenta. Since we expect that any reasonable quantum process could not produce a universe with non vanishing energy, we have called these universes with vanishing energy and momenta *creatable* universes. In the above quoted papers, the authors assume that energy and momenta can only be properly defined

in asymptotic Minkowskian universes and, consequently, they could not follow our line of research. Nevertheless, in the present paper, we have been able to define the energy and momenta of some non asymptotic Minkowskian universes in a consistent and unambiguous way (the basic idea supporting our procedures). Asymptotically, perturbed and non perturbed FRW universes appear to be conformally flat, and this fact has allowed a definition of energy and momenta which is a generalization of that used in the case of asymptotic Minkowskian space-times. In both cases, we are constrained to calculate the energy and momenta in appropriate coordinate systems making explicit a certain form of the spatial metric and, then, showing how coordinate transformations preserving this form do not alter the energy and momenta.

Furthermore, whereas in papers [6]–[9] only exact FRW universes were studied, we have considered realistic perturbed FRW universes (a related study considering departures from FRW geometries was proposed in [6]). In this way, we have found that perturbed closed FRW universes are creatable, but perturbed flat FRW universes, in the framework of standard inflation, are not.

Notice that, attending the different criteria put forward to define a universe as creatable (to have a finite tunneling amplitude, in the case of the quoted authors, or to have vanishing energy and momenta, in our case), it is not obvious that both definitions must lead to the same conclusions. Comparison is possible for unperturbed FRW models (the quoted authors have not studied the perturbed ones). There is full agreement in the closed case, but for non perturbed flat models, there is some discrepancy, since the quoted authors find them creatable only in the de Sitter subcase, whereas we find all them creatable. Nevertheless, this discrepancy has perhaps a non significant meaning, since when we have considered a realistic perturbed flat FRW model, we have found that this model is not creatable. This result shows that the exact flat FRW universe is an unstable one to our concerning. In the case of perturbed FRW universes, the comparison with the results of these authors is not possible since, as we have said, they have not considered the case.

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APPENDIX: SCALAR AND TENSOR INFLATIONARY PERTURBATIONS IN A FLAT BACKGROUND

Our protocol begins with the use of Gauss coordinates, in which, the metric has the form:

$$ds^2 = -dt^2 + dl^2, \quad dl^2 = g_{ij}dx^i dx^j, \quad (\text{A.1})$$

this means that, in Gauss coordinates, the conditions

$$g_{00} = -1, \quad g_{0i} = 0 \quad (\text{A.2})$$

are satisfied. In the case of a FRW universe with scalar perturbations, conditions (A.2) define the so-called synchronous gauge. In this gauge, a detailed study about the evolution of scalar perturbations in a flat FRW universe can be found in reference [13]. The authors of that paper (Ma and Bertschinger) studied scalar perturbations in a rather general FRW flat universe containing baryons, cold dark matter (CDM), neutrinos, and radiation. They also studied the evolution in the longitudinal gauge. In the synchronous gauge, Ma and Bertschinger expanded the most general scalar metric perturbation as it is done in Eq. (7). This expansion only involves two arbitrary functions: $\eta(\mathbf{k}, \tau)$ and $h(\mathbf{k}, \tau)$.

In the longitudinal gauge one can write:

$$ds^2 = a^2(\tau)[-(1 + 2\psi)d\tau^2 + (1 + 2\phi)\delta_{ij}dx^i dx^j], \quad (\text{A.3})$$

where only two arbitrary functions are necessary to describe the most general scalar perturbation. Fourier expansions of ψ and ϕ involve the coefficients $\psi(\mathbf{k}, \tau)$ and $\phi(\mathbf{k}, \tau)$.

There are other coefficients appearing in the expansions of physical quantities involved in the energy momentum tensor, e. g., those corresponding to the density contrasts of the different energy components: $\delta_\gamma(\mathbf{k}, \tau)$ for radiation, $\delta_c(\mathbf{k}, \tau)$ for CDM and so on. All these coefficients are coupled in a complicate system of equations (see [13]).

Fortunately, in order to evaluate the integrals giving the total energy and momenta of the perturbed universe, only functions $\eta(\mathbf{k}, \tau)$ and $h(\mathbf{k}, \tau)$ (related to the metric) could be actually necessary; this fact facilitates our calculations. Moreover, only functions $\eta(\mathbf{k}, \tau_0)$ and $h(\mathbf{k}, \tau_0)$ are required, τ_0 being an arbitrary time. We can say that our problem is identical

to that solved in paper [13], where initial conditions to solve the equations governing the evolution of the perturbations were calculated at a fixed time. This time was chosen to be in the time interval limited by electron positron annihilation and the time at which light massive neutrinos become non relativistic. The same choice is appropriate for us. Ma and Bertschinger solved the evolution equations in the mentioned time interval for linear adiabatic perturbations larger than the horizon ($k\tau \ll 1$). In the synchronous gauge, these authors found:

$$\eta = 2C + \frac{5 + 4R_\nu}{6(15 + 4R_\nu)} C k^2 \tau^2, \quad h = C k^2 \tau^2, \quad (\text{A.4})$$

where $C = C(\mathbf{k})$, $R_\nu = \bar{\rho}_\nu / (\bar{\rho}_\gamma + \bar{\rho}_\nu)$, and $\bar{\rho}_\nu$ and $\bar{\rho}_\gamma$ are the background energy densities of neutrinos and photons, respectively. There is also a time decaying term whose explicit form is given in Eq. (20).

The same study was also performed in the longitudinal gauge. Indeed, Ma and Bertschinger fixed their initial conditions by assuming that, in the longitudinal gauge, under the assumption of statistical isotropy, the power spectrum of the ψ potential of Eq. (A.3) is

$$P(\psi) \propto k^{-3}, \quad (\text{A.5})$$

(see below for comments about this choice). How can we obtain the corresponding initial conditions in the synchronous gauge? The answer is easily obtained from Eqs. (18) in [13]. One of these equations reads as follows:

$$\psi(\mathbf{k}, \tau) = \frac{1}{2k^2} \left(\ddot{h}(\mathbf{k}, \tau) + 6\ddot{\eta}(\mathbf{k}, \tau) + \tau^{-1} [\dot{h}(\mathbf{k}, \tau) + 6\dot{\eta}(\mathbf{k}, \tau)] \right), \quad (\text{A.6})$$

where each dot stands for a derivative with respect to the conformal time.

Taking into account Eqs. (A.4)–(A.5) and this last equation, one easily gets

$$C(k) \propto k^{-3/2}, \quad (\text{A.7})$$

which is valid in the synchronous gauge. That could be our basic condition in order to compute the integrals giving the total energy and momenta of the universe. It is also the basic assumption leading to the initial conditions used by Ma and Bertschinger to solve the evolution equations in the synchronous gauge. The resulting numerical solution gave a very good description of both the power spectrum, $P(k)$, of the energy density perturbations and the angular power spectrum (C_ℓ coefficients) of the CMB.

Now, a question arises: why the ψ -spectrum defined in Eq. (A.5) is appropriate? Let us try to answer this.

After evolution, large enough cosmological inhomogeneities reenter the horizon in the matter dominated era and, afterward, it is well known that the potential ψ satisfies the equation (see [13])

$$\Delta\psi \propto \delta . \quad (\text{A.8})$$

Since δ is the total energy density contrast, this last equation indicates that function ψ plays the role of the peculiar Newtonian gravitational potential. Finally, Eqs. (A.5) and (A.8) lead to

$$P(k) = \langle |\delta_{\mathbf{k}}|^2 \rangle \propto k , \quad (\text{A.9})$$

which means that the spectrum of the energy density perturbations is a Harrison-Zel'dovich (HZ) one. This result justifies the use of the ψ -spectrum defined in Eq. (A.5). The above HZ spectrum is only valid at times close enough to horizon crossing, but afterward, as the inhomogeneities evolve inside the horizon, microphysics becomes important and this spectrum evolves toward a new one of the form $P(k) = k/T(k)$, where $T(k)$ is the so-called transfer function.

Inflationary predictions are compatible with a HZ spectrum $P(k) \propto k$, as well as with an spectrum of the form $P(k) \propto k^{n_s}$ having its spectral index n_s close to unity. Accordingly, the analysis of the data obtained by the WMAP mission during three observation years leads to the inequality $0.942 < n_s < 0.974$. Moreover, if other observational data (galaxy surveys, other CMB observations, and so on) are taken into account, the resulting inequality appears to be $0.932 < n_s < 0.962$ (see [16] and [21]). From Eq. (A.8) one easily proves that the condition $C(k) \propto k^{(n_s-4)/2}$ leads to a final spectrum $P(k) = k^{n_s}$. On account of these considerations, our calculations of the total energy and momenta of the universe is based on the relation:

$$C(k) \propto k^{(n_s-4)/2}, \quad n_s < 1, \quad n_s \simeq 1 , \quad (\text{A.10})$$

which coincides with Eq. (A.7) for $n_s = 1$.

We must emphasize that Eqs. (A.4) and (A.10) are only valid for adiabatic scalar perturbations evolving outside the horizon; hence, these relations only hold for $k < k_{max}$, where $k_{max} = 2\pi/L_0$ and $L_0 = H^{-1}(\tau_0)$; here, L_0 is the horizon size at the conformal time τ_0 . Moreover, if the adiabatic perturbations are inflationary, another inequality, $k > k_{min}$, must

be also satisfied, where $k_{min} = 2\pi/L_{I0}$ and L_{I0} is the size, at time τ_0 , of a region comparable to the effective horizon at the beginning of inflation; that is to say, L_{I0} is the typical size of the huge inflationary bubbles. For $k > k_{max}$ the perturbations evolve inside the horizon (where microphysics is important) and, consequently, the spectra of super-horizon perturbations must be modified by means of transfer functions. If these perturbations are inflationary, they are initially Gaussian and afterward, during the radiation dominated era (in particular, in the period considered in paper [13] and also in Sec. III A), they keep Gaussian because nonlinear processes leading to deviations from Gaussianity had not developed yet. Moreover, in the interval $k_{min} < k < \infty$, we assume that the perturbations produced during inflation are absolutely dominant against possible residual pre-inflationary fluctuations. The main reason is that, at the end of inflation, inflationary supercooling had made the pre-inflationary radiation density negligible against the total energy of the inflationary field and, consequently, after reheating, the mean radiation energy is fully dominated by the energy coming from the mean inflationary field and, evidently, the resulting adiabatic perturbations are associated to the fluctuations of this dominant field, with negligible contributions from pre-inflationary supercooled sources. Finally, for $k < k_{min}$, the perturbations are so long that they will have a pre-inflationary origin without any inflationary contribution. Thus, though these pre-inflationary perturbations can be expected to be small, their contribution to the integral in Eq. (12) could be significant as k tends to zero and r tends to infinity. Thus, as a precaution, this interval has been also considered along the paper. Of course, it has been taken into account that these pre-inflationary perturbations could be non Gaussian. All these ideas are carefully taken into account in Secs. III A and IV.

In general, inflation produces both scalar and tensor perturbations of the background universe. Some general considerations about tensor perturbations are now worthwhile. The tensor metric perturbations of a flat universe can be written in the form:

$$h_{ij}^T(\mathbf{x}, \tau) = \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} h_{ij}^T(\mathbf{k}, \tau) = \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} H_T(\mathbf{k}, \tau) \epsilon_{ij}(\hat{\mathbf{k}}) , \quad (\text{A.11})$$

where functions ϵ_{ij} satisfy the following equations:

$$\epsilon_{ij} = \epsilon_{ji}, \quad \epsilon_{ii} = 0, \quad \epsilon_{ij} k_i = 0 , \quad (\text{A.12})$$

which ensure that quantities $h_{ij}^T(\mathbf{x}, \tau)$ are symmetric, traceless, and divergenceless, as it must be in the case of metric perturbations describing gravitational waves. It is noticeable that

functions ϵ_{ij} only depend on the unit vector $\hat{\mathbf{k}}$ and, consequently, any dependence on the scale (on k) of the tensor metric perturbation is involved in the coefficient $H_T(\mathbf{k}, \tau)$. This scale dependence is usually fixed by defining a new power spectrum ([22])

$$P_T(k) = k^3 \langle |H_T(k)|^2 \rangle \propto k^{n_T} , \quad (\text{A.13})$$

where n_T is the tensor spectral index. With this spectrum, quantities $h_{ij}^T(\mathbf{k})$ are proportional to $k^{(n_T-3)/2}$. Taking into account previous formulas and assumptions, we could calculate the total energy and momenta of the universe for different n_T values. The explicit form of functions ϵ_{ij} is not necessary. Nevertheless, an explicit representation of these quantities can be easily found from the definitions given in [23]. What can we say about the spectral index n_T ?

The spectrum generated by most inflationary potentials has an spectral index $n_T \simeq 1 - n_s$ (see reference [24]); hence, from previous comments about n_s -values and WMAP data, it follows that the power spectrum of these inflationary backgrounds of gravitational waves is very flat (small but non vanishing n_T value). In the presence of these gravitational waves, the spectral index n_s deviates from unity; by this reason and with the essential aim of allowing the existence of an inflationary background of gravitational waves (which is studied in Sec. III B), the n_s values used in Sec. III A have been assumed to be slightly smaller than unity.

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